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Non-renormalization theorems without supergraphs: The Wess–Zumino Model

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Abstract

The non-renormalization theorems of chiral vertex functions are derived on the basis of an algebraic analysis. The property, that the interaction vertex is a second supersymmetry variation of a lower dimensional field monomial, is used to relate chiral Green functions to superficially convergent Green functions by extracting the two supersymmetry variations from an internal vertex and transforming them to derivatives acting on external legs. The analysis is valid in the massive as well as in the massless model and can be performed irrespective of properties of the superpotential at vanishing momentum.

1. Introduction

Cancellations of ultraviolet divergencies in globally supersymmetric field theories, sometimes denoted as mysterious, have been observed at a very early stage in the development of the field [1] and have been ever since considered as a hallmark of these theories. Iliopoulos and Zumino [2] were the first to analyze such cancellations, which are not (direct) consequences of the supersymmetric Ward identities, in the framework of supersymmetric Wess–Zumino models. Since then several authors [3, 4, 5] have contributed to the development and streamlining of supergraph formalism in superspace, by which the cancellations in various models can be verified straightforwardly. A careful examination on the other hand of the renormalization of supersymmetric field theories by means of the algebraic renormalization procedure has been undertaken in [6]. But the cancellations of ultraviolet singularities, alias the non-renormalization theorems, were rather elusive in this approach. Our purpose is to overcome this shortcoming by a derivation of the non-renormalization theorems adaptable to the algebraic renormalization method. As a first step into this direction we reconsider here the d=4 Wess–Zumino model. We will show that the non-renormalization theorems in this model can be deduced from the fact that the interaction Lagrangian of mass dimension 4 may be represented as second supersymmetry variation of a field monomial with mass dimension 3:

$$F\varphi^2 - \frac{1}{2}\varphi\psi^2 = \frac{1}{12}\delta^Q_\alpha \delta^{Q\alpha}\varphi^3, \qquad \bar{F}\bar{\varphi}^2 - \frac{1}{2}\bar{\varphi}\bar{\psi}^2 = \frac{1}{12}\delta^{\bar{Q}\dot{\alpha}}\delta^{\bar{Q}}_{\dot{\alpha}}\bar{\varphi}^3. \tag{1.1}$$

Here δ^Q_α and $\delta^{\bar{Q}}_{\dot{\alpha}}$ are the supersymmetry transformations and (φ, ψ, F) is the chiral field multiplet and $(\bar{\varphi}, \bar{\psi}, \bar{F})$ is the antichiral field multiplet.

This sort of cohomological argument was first used by Zumino [7] to demonstrate that the vacuum graphs in supersymmetric field theories vanish. It was later on exploited [8] for the construction of the perturbative (inverse) Nicolai map.

Our argumentation will rely on two ingredients. First of all we extend the Wess–Zumino model, coupling the chiral and anti-chiral interaction vertices to external field multiplets. Local couplings have been already considered in the context of conformal symmetry breaking in the ordinary φ^4 model [9]. In the Wess–Zumino model local couplings are necessarily superfields and in this way the cohomological structure of the interaction vertices (1.1) is included in the action of the extended Wess–Zumino model. The original model is recovered in the limit of constant external field. The non-renormalization theorems will appear as simple consequence of the supersymmetry Ward identities of the extended model. We make secondly use of a modified R-symmetry of the extended model involving a non-trivial R-transformation of the external field multiplet. The modified R-symmetry will turn out to be an efficient bookkeeping device for our purposes.

The massless Wess–Zumino model requires a separate consideration. It was noted in Refs. [10, 11] that in contradiction to standard lore the radiative corrections to the superpotential of the massless Wess–Zumino model do not vanish. Deriving the non-renormalization theorems from the cohomological structure of the interaction vertices, it is possible to disentangle the ultraviolet and infrared aspects of this phenomenon. The supersymmetric specific ultraviolet cancellation based on eq. (1.1) survives the massless limit. The integrated $\bar{\varphi}^3$-insertion of mass dimension 3 on the other hand develops an infrared singularity at zero momentum which is canceled by the explicitly extracted powers of external momenta leaving behind a non-vanishing contribution to the effective potential.

In section 2 we introduce local superfield couplings in the Wess–Zumino model and define in this way the Green functions with covariant vertex insertions. From the supersymmetry Ward identities the non-renormalization theorem for chiral Green functions with one antichiral vertex insertion is derived. In section 3 we discuss consequences of the modified R-symmetry and supersymmetry for 1PI Green functions. In section 4 we relate the chiral Green functions with antichiral vertex insertions to the chiral Green functions of the Wess–Zumino model. We profit from the modified R-symmetry, which simplifies the combinatorics and makes possible to generalize the result to arbitrary normalization conditions of the wave-function renormalization. In section 5 we treat the massless model and discuss the limit to vanishing momenta. In an appendix we give the results of the algebraic analysis of non-renormalization theorems in expressions of component fields. Throughout the paper we will use the notations of Ref. [6].

2. The Wess–Zumino model with local couplings

The Wess–Zumino model contains the chiral superfield $A(x,\theta,\bar{\theta})$ and its complex conjugate, the antichiral superfield $\bar{A}(x,\theta,\bar{\theta})$. In the chiral and antichiral representation they are expanded in the component fields according to

$$A(x,\theta) = \varphi(x) + \theta\chi(x) + \theta^2 F(x), \qquad \bar{A}(x,\bar{\theta}) = \bar{\varphi}(x) + \bar{\theta}\bar{\chi}(x) + \bar{\theta}^2 \bar{F}(x). \tag{2.1}$$

The classical action[1]

$$\Gamma_{cl} = \frac{1}{16}\int dV\, A e^{2i\theta\sigma\bar{\theta}\partial}\bar{A} + \frac{1}{4}\int dS\Big(\frac{m}{2}A^2 + \frac{\lambda}{12}A^3\Big) + \frac{1}{4}\int d\bar{S}\Big(\frac{m}{2}\bar{A}^2 + \frac{\lambda}{12}\bar{A}^3\Big) \tag{2.2}$$

[1] Throughout the paper we take chiral fields in the chiral and antichiral fields in antichiral representation.

is invariant under supersymmetry transformations:

$$\delta^Q_\alpha A = \frac{\partial}{\partial\theta^\alpha} A \qquad \delta^Q_\alpha \bar{A} = 2i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu\bar{A} \tag{2.3}$$
$$\delta^{\bar{Q}}_{\dot{\alpha}} A = -2i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu A \qquad \delta^{\bar{Q}}_{\dot{\alpha}}\bar{A} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\bar{A}$$

The Wess–Zumino model is renormalizable and the Ward identities of supersymmetry,

$$\mathcal{W}^Q_\alpha\Gamma(A,\bar{A}) = 0 \quad \text{and} \quad \mathcal{W}^{\bar{Q}}_{\dot{\alpha}}\Gamma(A,\bar{A}) = 0, \tag{2.4}$$

define the Green function to all orders of perturbation theory. $\Gamma(A,\bar{A})$ denotes the generating functional of one particle irreducible (1PI) Green functions. Free parameters are fixed by the following normalization conditions:

$$\Gamma_{F\bar{F}}(p^2)\Big|_{p^2=\kappa^2} = 1 \tag{2.5a}$$
$$\Gamma_{F\varphi}(p^2)\Big|_{p^2=0} = m \tag{2.5b}$$
$$\Gamma_{F\varphi\varphi}(p_1,p_2,p_3)\Big|_{p_i=0} = -\frac{\lambda}{2} \tag{2.5c}$$

The wave-function renormalization is fixed at an arbitrary normalization point κ^2, whereas the mass and the coupling are normalized at $p_i = 0$. When the model is constructed with local couplings in section 3 and section 4, it is seen, that the normalization point $p_i = 0$ for chiral vertices is indeed distinguished from other normalization momenta. In this paper we refer to the BPHZ scheme [12] in superspace [13] for removing the divergencies. It has the advantage to preserve supersymmetry in the subtraction procedure. The subtraction degree and the possible degree of divergence are given by simple power counting theorems in superspace. In particular the maximal degree of divergence for a supergraph with $N_S = N_A + N_{\bar{A}}$ external legs is determined by

$$d_\Omega \le 3 - N_S \quad \text{if the supergraph has only chiral or antichiral legs,} \tag{2.6}$$
$$d_\Omega \le 2 - N_S \quad \text{if chiral and antichiral legs are present.}$$

By power counting all diagrams belonging to vertices of the classical action could be logarithmically divergent. The non-renormalization theorems of chiral vertex functions, i.e. the actual finiteness of all chiral and antichiral vertex functions, are not an immediate consequence of supersymmetry, but are proven by an explicit evaluation of diagrams in superspace [3, 4, 5].

In the present paper we derive the non-renormalization theorems of chiral vertices from the algebraic property, that the interaction vertices of the Wess–Zumino model are supersymmetry variations of lower dimensional field monomials:

$$\begin{aligned} F\varphi^2 - \frac{1}{2}\varphi\psi^2 &= \frac{1}{4}\delta^Q_\alpha(\varphi^2\psi^\alpha) = \frac{1}{12}\delta^Q_\alpha\delta^{Q\alpha}\varphi^3, \\ \bar F\bar\varphi^2 - \frac{1}{2}\bar\varphi\bar\psi^2 &= \frac{1}{4}\delta^{\bar Q\dot\alpha}(\varphi^2\bar\psi_{\dot\alpha}) = \frac{1}{12}\delta^{\bar Q\dot\alpha}\delta^{\bar Q}_{\dot\alpha}\bar\varphi^3. \end{aligned} \tag{2.7}$$

In terms of superfields these properties are summarized in the supersymmetry transformations:

$$\begin{aligned} \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\alpha}A^3 = \delta^{Q\alpha}\frac{\partial}{\partial\theta^\alpha}A^3 = \delta^{Q\alpha}\delta^Q_\alpha A^3, \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar A^3 = -\delta^{\bar Q}_{\dot\alpha}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar A^3 = \delta^{\bar Q}_{\dot\alpha}\delta^{\bar Q\dot\alpha}\bar A^3. \end{aligned} \tag{2.8}$$

When one wants to consider the effect of this property on Green functions, supersymmetric covariant vertices $A^3(x,\theta)$ and $\bar A^3(x,\bar\theta)$ instead of the invariant vertices $\int dS\ A^3(x,\theta)$ and $\int d\bar S\ \bar A^3(x,\bar\theta)$ have to be included into the construction of the Wess–Zumino model.

The construction of covariant insertions in quantum field theory is a well understood subject and is most easily performed by coupling the covariant field polynomials to external fields in such a way, that the complete action is invariant. Accordingly we introduce external chiral and antichiral superfields with dimension zero,

$$\Lambda(x,\theta) = \lambda(x) + \theta\chi(x) + \theta^2 f(x), \qquad \bar\Lambda(x,\theta) = \bar\lambda(x) + \bar\theta\bar\chi(x) + \bar\theta^2\bar f(x), \tag{2.9}$$

which transform under supersymmetry transformations as the fields A and $\bar A$ (2.3). Since the field monomials A^3 and $\bar A^3$ are transformed as the fields, supersymmetry is maintained, when we couple Λ and $\bar\Lambda$ to the covariant insertions:

$$A^3(x,\theta) \longrightarrow \int dS\Lambda(x,\theta)A^3(x,\theta), \qquad \bar A^3(x,\bar\theta) \longrightarrow \int d\bar S\bar\Lambda(x,\bar\theta)\bar A^3(x,\bar\theta).$$

Instead of adding a further interaction term to the action of the Wess–Zumino model we interpret the fields $\Lambda(x,\theta)$ and $\Lambda(x,\bar\theta)$ as local couplings in superspace:

$$\Gamma_{cl} = \frac{1}{16}\int dV\ Ae^{2i\theta\sigma\bar\theta\partial}\bar A + \frac{1}{4}\int dS\ \big(\frac{m}{2}A^2 + \frac{1}{12}\Lambda A^3\big) + \frac{1}{4}\int d\bar S\ \big(\frac{m}{2}\bar A^2 + \frac{1}{12}\bar\Lambda\bar A^3\big). \tag{2.10}$$

The classical action of Wess–Zumino model (2.2) is recovered by taking the limit

$$\Lambda(x,\theta) \to \lambda \qquad \text{and} \qquad \bar\Lambda(x,\bar\theta) \to \lambda.$$

Contributions to Green functions with local couplings are defined in higher orders as usual by the Gell-Mann–Low formula, Wick's theorem and a subtraction scheme for removing the divergencies. The BPHZ scheme in superspace can be applied to the Wess–Zumino model with local coupling without modifications. In this scheme the Ward identities of supersymmetry are fulfilled to all orders by construction:

$$\mathcal{W}^Q_\alpha \Gamma(A,\bar A,\Lambda,\bar\Lambda) = 0 \ , \qquad \mathcal{W}^{\bar Q}_{\dot\alpha} \Gamma(A,\bar A,\Lambda,\bar\Lambda) = 0 \tag{2.11}$$

with

$$\begin{aligned}
\mathcal{W}^Q_\alpha &= \int dS \,\Big(\frac{\partial}{\partial\theta^\alpha} A \frac{\delta}{\delta A} + \frac{\partial}{\partial\theta^\alpha}\Lambda\frac{\delta}{\delta\Lambda}\Big) + \int d\bar S\,\Big(2i(\sigma^\mu\bar\theta)_\alpha\big(\partial_\mu \bar A\frac{\delta}{\delta\bar A} + \partial_\mu\bar\Lambda\frac{\delta}{\delta\bar\Lambda}\big)\Big) \\
\mathcal{W}^{\bar Q}_{\dot\alpha} &= \int d\bar S\,\Big(-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar A\frac{\delta}{\delta\bar A} - \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\Lambda\frac{\delta}{\delta\bar\Lambda}\Big) + \int d\bar S\,\Big(-2i(\theta\sigma^\mu)_{\dot\alpha}\big(\partial_\mu A\frac{\delta}{\delta A} + \partial_\mu\Lambda\frac{\delta}{\delta\Lambda}\big)\Big)
\end{aligned} \tag{2.12}$$

$\Gamma = \Gamma(A,\bar A,\Lambda,\bar\Lambda)$ is the generating functional of 1PI Green functions in the Wess Zumino model with local field couplings. Its lowest order $\Gamma^{(0)}(A,\bar A,\Lambda,\bar\Lambda)$ is defined by the classical action (2.10). In loop order $L\geq 1$ it is an expansion in external fields $\Lambda,\bar\Lambda$ and fields $A,\bar A$:

$$\begin{aligned}
&\Gamma^{(L)}(A,\bar A,\Lambda,\bar\Lambda) \\
= &\sum_n\sum_{\bar n}\sum_{m=n}^{n+2(L-1)}\prod_{k=1}^{n}\int dS_k A(z_k)\prod_{l=1}^{\bar n}\int dS_l\bar A(\bar z_l)\prod_{i=n+1}^{m+n}\int dS_i\Lambda(z_i)\prod_{j=\bar n+1}^{\bar n+\bar m}\int dS_j\Lambda(\bar z_j) \\
&\qquad\frac{1}{n!m!\bar n!\bar m!}\Gamma^{(L)}_{n,\bar n,m,\bar m}(z_1,\dots z_{m+n},\bar z_1,\dots \bar z_{\bar m+\bar n})
\end{aligned} \tag{2.13}$$

with $z_i\equiv(x_i,\theta_i)$ and $\bar z_j\equiv(\bar x_j,\bar\theta_j)$. Since the perturbative expansion is an expansion in the local couplings, the total number of chiral and antichiral vertices is determined by the number of A- and $\bar A$-legs and increases by 2 from order to order: the number $\bar m$ of antichiral $\bar\Lambda$'s is therefore not an independent quantity in eq. (2.13):

$$\bar m \equiv \bar m(n,\bar n,m,L) = n+\bar n+2(L-1)-m. \tag{2.14}$$

The generating functional of 1PI Green functions of the ordinary Wess–Zumino model is obtained in the limit to constant coupling:

$$\lim_{\Lambda,\bar\Lambda\to\lambda}\Gamma(A,\bar A,\Lambda,\bar\Lambda) = \Gamma(A,\bar A). \tag{2.15}$$

For defining the Green functions of the extended Wess–Zumino model unambiguously the possible counterterms depending on local couplings have to be fixed by suitable normalization conditions and symmetries. We postpone this discussion to the next section and deduce here immediately the most important results of the construction. For this purpose the normalization conditions are implicitly defined by the BPHZ scheme at zero momentum.

Differentiating the generating functional with local couplings (2.13) with respect to Λ or $\bar\Lambda$ and taking the limit to constant coupling it is possible study vertex functions of the Wess–Zumino model with one insertion of a chiral or antichiral supersymmetric covariant vertex:

$$\lim_{\Lambda,\bar\Lambda\to\lambda}\frac{\delta}{\delta\Lambda(x,\theta)}\Gamma(A,\bar A,\Lambda,\bar\Lambda)=\frac{1}{48}[A^3(x,\theta)]_3\cdot\Gamma(A,\bar A) \tag{2.16}$$
$$\lim_{\Lambda,\bar\Lambda\to\lambda}\frac{\delta}{\delta\bar\Lambda(\bar x,\bar\theta)}\Gamma(A,\bar A,\Lambda,\bar\Lambda)=\frac{1}{48}[\bar A^3(\bar x,\bar\theta)]_3\cdot\Gamma(A,\bar A)$$

In the BPHZ scheme the degree of divergence of a Green function with insertion $[Q]_d$ is determined by the (minimal) subtraction degree d of the vertex and the dimensions of external legs. In superspace θ's are counted with dimension $-\frac{1}{2}$ and can be extracted from the insertion by raising its dimensions by $+\frac{1}{2}$. (See Ref. [6] for a detailed definition of normal products in superspace.) Since the model with local couplings satisfies the supersymmetry Ward identities (2.11) one derives consequences for the Green functions with covariant vertex insertions by differentiating the Ward identity with respect to $\bar\Lambda$:[2]

$$\mathcal{W}^{\bar Q}_{\dot\alpha}\frac{\delta}{\delta\bar\Lambda(x,\bar\theta)}\Gamma \;=\; -\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\delta}{\delta\bar\Lambda(x,\bar\theta)}\Gamma. \tag{2.17}$$

In the limit to constant λ one derives the Ward identity of supersymmetric covariant vertex insertions:

$$\mathcal{W}^{\bar Q}_{\dot\alpha}\Big([\bar A^3(x,\bar\theta)]_3\cdot\Gamma\Big) \;=\; -[\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar A^3(x,\bar\theta)]_{\frac{5}{2}}\cdot\Gamma \tag{2.18}$$

(The Ward operator of supersymmetry on the left-hand-side only involves the fields A and $\bar A$.) Eq. (2.18) is the key relation for the foundation of the non-renormalization theorems, since it generalizes the cohomological property, that invariant vertices are supersymmetry variations, eq. (2.8), to insertions into nonlocal diagrams. Differentiating eq. (2.18) once more with respect to $\bar\theta$, one is able to derive an identity, which relates the highest 4-dimensional component of the supermultiplet $\bar A^3$ with its lowest 3-dimensional component by applying twice the Ward operator of supersymmetry transformations:

$$\mathcal{W}^{\bar Q}_{\dot\alpha}\mathcal{W}^{\bar Q\dot\alpha}\Big([\bar A^3(x,\bar\theta)]_3\cdot\Gamma\Big)=[\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar A^3(x,\bar\theta)]_4\cdot\Gamma \tag{2.19}$$

Evaluating this identity for n external chiral $A(x_i,\theta_i)$-legs one derives consequences for chiral vertex functions with an antichiral vertex insertion:

$$\begin{aligned}&-\Big([\frac{1}{4}(\bar F\bar\varphi^2-\frac{1}{2}\bar\varphi\bar\psi^2)]_4\cdot\Gamma\Big)_n(x;x_1,\theta_1,\dots x_n,\theta_n) \\ &=\sum_{k=1}^n\sum_{l=1}^n\Big(\theta_k\theta_l\eta^{\mu\nu}-i\theta_k\sigma^{\mu\nu}\theta_l\Big)\frac{\partial}{\partial x_k^\mu}\frac{\partial}{\partial x_l^\nu}\Big(\big[\frac{1}{12}\bar\varphi^3\big]_3\cdot\Gamma\Big)_n(x;x_1,\theta_1,\dots x_n,\theta_n)\end{aligned} \tag{2.20}$$

[2]From now on we restrict the discussion to insertions of antichiral vertices. The analog results for chiral insertions are derived by complex conjugation.

Integrating over x and using translational invariance chiral vertex functions with an integrated internal (supersymmetric) antichiral vertex are related to vertex functions with a 3-dimensional integrated $\bar\varphi^3$-insertion and two explicit x-derivatives:

$$
\begin{aligned}
&-\Big(\big[\int d^4x\ \frac{1}{4}(\bar F\bar\varphi^2-\frac{1}{2}\bar\varphi\bar\psi^2)\big]_4\cdot\Gamma\Big)_n(x_1,\theta_{1n},\dots x_n) \qquad (2.21)\\
&=\sum_{k=1}^{n}\sum_{l=1}^{n}\Big(\theta_{kn}\theta_{ln}\eta^{\mu\nu}-i\theta_{kn}\sigma^{\mu\nu}\theta_{ln}\Big)\frac{\partial}{\partial x_k^\mu}\frac{\partial}{\partial x_l^\nu}\Big(\big[\int d^4x\ \frac{1}{12}\bar\varphi^3\big]_3\cdot\Gamma\Big)_n(x_1,\theta_{1n},\dots x_n)
\end{aligned}
$$

It is seen from this equation that chiral vertex functions with one antichiral vertex vanish at zero momentum and by power counting the degree of divergence is improved by 2 from the left-hand-side to the right-hand-side: One gets one degree of improvement from the dimension of the 3-insertion and one degree from two θ variables counting each with dimension $\frac{1}{2}$. For the specific normalization conditions of the BPHZ scheme at zero momentum eq. (2.21) already implies the non-renormalization theorems of chiral vertex functions in the Wess–Zumino model. Since any chiral vertex function necessarily includes antichiral vertices, the 1PI Green functions of the Wess–Zumino model can be composed from Green functions with integrated antichiral vertex insertions, if one takes into account the correct combinatorial factors. Due to a new R-symmetry of the extended Wess–Zumino model it turns out that the combinatorics indeed becomes very simple, since in each loop order the number of antichiral vertices in the contributing diagrams is fixed. In section 4 we complete the above argumentation and generalize the result in such a way, that also non-trivial normalization conditions of the wave-function renormalization are taken into account.

For completeness we want to mention that on the basis of (2.18) and (2.19) one can study insertions of antichiral vertices in Green functions with antichiral legs or with chiral and antichiral legs. In both cases the power counting degree is not improved by the relations of the covariant insertions since in the supersymmetry transformations no x-derivatives are extracted, but the dimension of external legs is lowered by two θ-differentiations.

3. R'-symmetry, supersymmetry and local couplings

The classical action of the Wess–Zumino model with local coupling (2.10) is also invariant under a R-symmetry,

$$\mathcal{W}^{R'}\Gamma_{cl} = 0, \tag{3.1}$$

whose generator is formally given by

$$\begin{aligned}\mathcal{W}^{R'} &= \int dS\, i(-A + \theta\frac{\partial}{\partial\theta}A)\frac{\delta}{\delta A} + \int dS\, i(\Lambda + \theta\frac{\partial}{\partial\theta}\Lambda)\frac{\delta}{\delta\Lambda} \\ &+ \int d\bar{S}\, i(\bar{A} + \bar{\theta}\frac{\partial}{\partial\bar{\theta}}\bar{A})\frac{\delta}{\delta\bar{A}} + \int d\bar{S}\, i(-\bar{\Lambda} + \bar{\theta}\frac{\partial}{\partial\bar{\theta}}\bar{\Lambda})\frac{\delta}{\delta\bar{\Lambda}}.\end{aligned} \tag{3.2}$$

(We call it R'-symmetry, in order to distinguish it from usual conformal R-symmetry of the massless Wess–Zumino model.) The generator (3.2) attaches an R'-weight -1 to the chiral fields A and a R'-weight $+1$ to the local coupling Λ with the corresponding sign reversal for the respective conjugate fields. θ's and $\bar{\theta}$'s are counted with a weight $+1$ and -1 respectively. R'-symmetry is naively maintained in the procedure of renormalization. The BPHZ scheme adopted further on does respect supersymmetry and so we proceed to derive consequences of those symmetries for the quantum action Γ (2.13):

$$\Gamma = \Gamma_{cl} + \sum_{L=1}^{\infty}\Gamma^{(L)} \tag{3.3}$$

The implementation of supersymmetry restricts the θ- and $\bar{\theta}$-dependence of the 1PI Green functions [3]:

$$\begin{aligned}&\Gamma^{(L)}_{n,\bar{n},m,\bar{m}}(x_1,\theta_1,\ldots x_{n+m},\theta_{n+m},\bar{x}_1,\bar{\theta}_1,\ldots \bar{x}_{\bar{n}+\bar{m}},\bar{\theta}_{\bar{n}+\bar{m}}) \\ &= \exp\Big(2i\sum_{i=1}^{m+n}\theta_i\sigma^{\mu}\bar{\theta}_{\bar{n}}\partial^i_{\mu} - 2i\sum_{j=1}^{\bar{n}+\bar{m}}\theta_n\sigma^{\mu}\bar{\theta}_{j\bar{n}}\partial^j_{\mu}\Big)F_{n,\bar{n},m,\bar{m}}(x_l,\theta_{ln},\bar{x}_{\bar{l}},\theta_{\bar{l}\bar{n}})\end{aligned} \tag{3.4}$$

with $\theta_{in} \equiv \theta_i - \theta_n$ and $\bar{\theta}_{j\bar{n}} \equiv \bar{\theta}_j - \bar{\theta}_{\bar{n}}$. (If only chiral or only antichiral legs and vertices are present, the exponent vanishes and the respective Green functions depend only on the differences θ_{in} or $\bar{\theta}_{j\bar{n}}$.)

We want to find out the restrictions due to R'-neutrality of the various contributions. Let us consider a superfield Green function with a fixed number, say x and $\bar{x}$, of external θ's and $\bar{\theta}$'s (figuring in the expansions of the fields A and Λ on one hand and of fields $\bar{A}$ and $\bar{\Lambda}$ on the other hand). The coefficient functions $F_{n,\bar{n},m,\bar{m}}$ can be expanded into a finite series of terms ordered with respect to the powers ω and $\bar{\omega}$ of the difference variables θ_{im} and $\theta_{j\bar{m}}$:

$$F_{n,\bar{n},m,\bar{m}} = \sum_{\omega=0}^{2(m+n-1)} \sum_{\bar{\omega}=0}^{2(\bar{n}+\bar{m}-1)} P^{\omega,\bar{\omega}}(\theta_{ln}, \bar{\theta}_{\bar{l}\bar{n}}) f_{\omega,\bar{\omega}}(x_l, \bar{x}_{\bar{l}}) \tag{3.5}$$

An additional source of θ's and $\bar{\theta}$'s is supplied by the exponential factor in (3.4). Let k be the (equal) number of θ's and $\bar{\theta}$'s to a specific term coming from the exponential. The overall number of θ's and $\bar{\theta}$'s (internal and external) is determined by the number of external legs and vertices. This yields

$$\omega + x + k = 2(m+n) \quad \text{and} \quad \bar{\omega} + \bar{x} + k = 2(\bar{m} + \bar{n}). \tag{3.6}$$

R'-neutrality, that is the vanishing of the R' weight of the specific contribution,

$$(\mathcal{W}^{R'}\Gamma)_{n,\bar{n},m,\bar{m}} = 0, \tag{3.7}$$

is guaranteed through the relation

$$n + \bar{n} + 3m - 3\bar{m} - \omega + \bar{\omega} = 0. \tag{3.8}$$

In addition we have the graphological constraint (2.14)

$$m + \bar{m} = n + \bar{n} + 2(L-1). \tag{3.9}$$

Eliminating in eqs. (3.6), (3.8) and (3.9) ω and $\bar{\omega}$ and solving for m and $\bar{m}$ one finds

$$\begin{aligned} m &= n + (L-1) + \frac{1}{2}(\bar{x} - x) \\ \bar{m} &= \bar{n} + (L-1) - \frac{1}{2}(\bar{x} - x). \end{aligned} \tag{3.10}$$

It means in words that the number of chiral and antichiral vertices is uniquely determined by the setting of external fields and the loop order. It is also easy to verify that the conditions (3.6), (3.8) and (3.9) cannot be matched non-trivially, if only chiral legs and chiral vertices are present ($\bar{n} = \bar{m} = 0$ and $\bar{\omega} = 0$) or if only antichiral legs and antichiral vertices are present ($n = m = 0$ and $\omega = 0$).

4. The non-renormalization of chiral vertex functions

For the consistent renormalization of the Wess-Zumino model with local couplings one has to fix all conceivable counterterms by suitable normalization conditions. In the BPHZ scheme possible counterterms are included in Γ_{eff}:

$$\Gamma_{eff} \;=\; \Gamma_o + \Gamma_{int} + \Gamma_{ct} \tag{4.1}$$

with

$$\begin{aligned}\Gamma_o &= \int dV\ Ae^{2i\theta\sigma\bar\theta\partial}\bar A + \frac{m}{8}\Big(\int dS\ A^2 + \int d\bar S\ \bar A^2\Big) \qquad (4.2)\\ \Gamma_{int} &= \frac{1}{48}\Big(\int dS\ \Lambda A^3 + \int d\bar S\ \bar\Lambda \bar A^3\Big)\end{aligned}$$

and Γ_{ct} denoting the most general collection of local counterterms (see below). The dimension of Γ_{eff} is restricted by renormalizability to be not larger than 4. Since the BPHZ scheme in superspace is a supersymmetric invariant scheme, the possible counterterms appearing in Γ_{eff} are supersymmetric. As we have shown in the last section R'-invariance excludes Green functions with exclusively consist of chiral vertices and chiral legs. Therefore it also forbids to add the corresponding trivial counterterms to the interaction vertex and to the mass term:

$$\mathcal{W}^{R'} \int dS\ \Lambda^n A^3 = i(n-1)\int dS\ \Lambda^n A^3 \neq 0,\ \text{if}\quad n>1, \tag{4.3a}$$

$$\mathcal{W}^{R'} \int dS\ \Lambda^n A^2 = in\int dS\ \Lambda^n A^2 \neq 0,\ \text{if}\quad n>0. \tag{4.3b}$$

Local field polynomials to the classical chiral vertex functions which include chiral Λ and antichiral $\bar\Lambda$ have dimension 5 and 6 and do not contribute to Γ_{eff}. Of course, this already expresses, that the corresponding diagrams are non-local. One remains with the counterterms to the kinetic term, whose dependence on Λ and $\bar\Lambda$ is again restricted by R'-neutrality (cf. (3.10) with $\bar x - x = 0$):

$$\Gamma_{ct} = \sum_{L=1}^{\infty}\int dV\ z^{(L)}\Lambda^L Ae^{2i\theta\sigma\bar\theta\partial}(\bar\Lambda^L\bar A) \tag{4.4}$$

We want to mention that there are also some further counterterms, which vanish in the limit to constant coupling. They are not relevant to the present analysis and we omit them from Γ_{eff}. The parameters $z^{(L)}$ may be fixed in loop order L by the normalization condition of the Wess-Zumino model (cf. (2.5)):

$$\lim_{\lambda(x),\bar\lambda(x)\to\lambda}\Gamma_{F\bar F}(p^2=\kappa^2)=1 \tag{4.5}$$

In order to complete the analysis of non-renormalization theorems of chiral vertex functions we consider 1PI Green functions with n external chiral A-legs and discuss the limit to constant coupling. Eventually the chiral Green functions of the Wess-Zumino model are expressed in terms of Green functions of the extended Wess-Zumino model, which are convergent by power counting. The manifest supersymmetric expressions for Green functions (3.4) with n chiral A-legs and m chiral and $\bar{m}$ antichiral vertices is given by

$$\begin{aligned}
&\Gamma^{(L)}_{n,0,m,\bar{m}}(x_1,\theta_1\dots x_{n+m},\theta_{n+m},\bar{x}_1,\bar{\theta}_1\dots\bar{x}_{\bar{m}},\bar{\theta}_{\bar{m}}) \qquad (4.6)\\
&= e^{2i\sum_{i=1}^{m+n}\theta_i\sigma^\mu\bar{\theta}_{\bar{m}}\partial^i_\mu - 2i\sum_{j=1}^{\bar{m}-1}\theta_n\sigma^\mu\bar{\theta}_{j\bar{m}}\partial^j_\mu} F_{n,0,m,\bar{m}}(x_1,\theta_{1m+n},\dots,x_{n+m},\bar{x}_1,\bar{\theta}_{1\bar{m}},\dots,\bar{x}_{\bar{m}}).
\end{aligned}$$

The power counting degree of divergence can be determined by the power counting formula in superspace [13, 6]

$$d_\Gamma = 4 - N_S - \sum_s N_s d_s + \frac{1}{2}(\omega+\bar{\omega}).$$

N_S is here the number of all chiral and antichiral legs,

$$N_S = N_A + N_\Lambda + N_{\bar{\Lambda}} = n + m + \bar{m} = 2n + 2(L-1), \tag{4.7}$$

and d_s denotes the dimension of the different fields,

$$\sum_s N_s d_s = N_A d_A + N_\Lambda d_\Lambda + N_{\bar{\Lambda}} d_{\bar{\Lambda}} = n. \tag{4.8}$$

ω and $\bar{\omega}$ are the degrees of θ's and $\bar{\theta}$'s in the power series expansion of the functions $F_{n,0,m,\bar{m}}$ (3.5):

$$d_{\Gamma_{n,0,m\bar{m}}} = 6 - 3n - 2L + \frac{1}{2}(\omega+\bar{\omega}) \le 2 - n \tag{4.9}$$

The limit to constant coupling is performed by integrating the m chiral and $\bar{m}$ antichiral vertices in superspace:

$$\begin{aligned}
&\Gamma^{(L)}_{n,0,m,\bar{m}}(x_1,\theta_1,\dots x_{n+m},\theta_{n+m},\bar{x}_1,\bar{\theta}_1,\dots\bar{x}_{\bar{m}},\bar{\theta}_{\bar{m}}) \stackrel{\Lambda,\bar{\Lambda}\to\lambda}{\longrightarrow} \qquad (4.10)\\
&\frac{1}{m!\bar{m}!}\int dS_{n+1}\dots\int dS_{n+m}\int d\bar{S}_1\dots\int d\bar{S}_{\bar{m}}\Gamma^{(L)}_{n,0,m,\bar{m}}(x_1,\theta_{1n+m},\dots x_{n+m},\bar{x}_1,\bar{\theta}_{1\bar{m}},\dots\bar{x}_{\bar{m}})\\
&= \frac{1}{m!\bar{m}!}\int dS_{n+1}\dots\int dS_{n+m}\int d\bar{S}_1\dots\int d\bar{S}_{\bar{m}} e^{2i\sum_{i=1}^{n-1}\theta_{in}\sigma\bar{\theta}_{\bar{m}}\partial^i} F_{n,0,m,\bar{m}}(x_1,\theta_{1n+m},\dots\bar{x}_{\bar{m}})
\end{aligned}$$

Of course in this limiting procedure the power counting degree (4.9) of the vertex functions is not changed.

For saturation of the $\bar{\theta}$-integrations in (4.10) the relevant superfield Green functions have to depend on $2\bar{m}$ $\bar{\theta}$'s. Since the functions $F_{n,0,m,\bar{m}}$ (4.6) only depend on the differences

$\bar{\theta}_{j\bar{m}}$, they contribute at most with $2(\bar{m}-1)$ $\bar{\theta}$'s. The expansion of the exponential function in (3.4) breaks off at second order, which yields the two further $\bar{\theta}$'s. Therefore we find, that all non-vanishing contributions have the following structure in the number of $\bar{\theta}$'s (cf. (3.6)):

$$\bar{\omega} = 2(\bar{m}-1), \qquad \bar{x} = 0, \qquad k = 2. \tag{4.11}$$

Let us now consider the number of θ's. The Green functions with $\omega = 2(2n+m-1)$ corresponding to $x = 0$ vanish in the limit to constant coupling. Before integration these Green functions are related to Green functions with n φ-legs and a supersymmetric vertex inserted:

$$\Big([\bar{F}\bar{\varphi}^2 - \frac{1}{2}\bar{\psi}^2\bar{\varphi}]_4 \cdot \Gamma\Big)_{\varphi\ldots\varphi}(x; x_1, \ldots x_n), \tag{4.12}$$

and are not present in the Wess-Zumino model due to supersymmetry. From R'-invariance (3.10) we read off that they correspond to diagrams with $n+L-1$ chiral vertices and $L-1$ antichiral vertices. Their power counting degree is determined from eq. (4.9) to be $2-n$.

The next less divergent type of superfield Green functions are the ones with $\omega = 2(n+m-2)$ corresponding to $x+k = 4$ (3.6). In loop order L they contain according to R'-neutrality (3.10) L antichiral vertices $\bar{\Lambda}$ and $n+L-2$ chiral vertices Λ. Since in the limit to constant coupling only the second order in the expansion of the exponential contributes, the resulting integrated vertex functions have $2(n-1)$ powers of θ's and are the chiral vertex functions Γ_n of the Wess-Zumino model:

$$\Gamma^{(L)}_{n,0,n+L-2,L} \stackrel{\Lambda,\bar{\Lambda}\to\lambda}{\longrightarrow} \Gamma^{(L)}_n \tag{4.13}$$

Their degree of divergence is determined from the power counting formula in presence of local field couplings, eq. (4.9). With

$$\frac{1}{2}(\omega+\bar{\omega}) = n+m+\bar{m}-3 = 2n+2(L-1)-3 \tag{4.14}$$

one has

$$d_{\Gamma_{n,0,n+L-2,L}} = d_{\Gamma_n} = 1-n. \tag{4.15}$$

Compared to power counting in the ordinary Wess-Zumino model (2.6) the degree of divergence is improved by 2 and all chiral Green functions of the Wess-Zumino model are convergent emerging from convergent diagrams of the extended Wess-Zumino model with local couplings. Evaluating (4.10) they are seen to vanish at zero momentum, since the second order of the exponential function yields two x-differentiations:

$$\begin{aligned}\Gamma_n^{(L)}(x_1,\theta_{1n},\dots x_n) &= \lambda^{n+2(L-1)}(\prod_{i=1}^{n-1}\theta_{in}^2)f_n^{(L)}(x_1,\dots x_n) \\ &= \lambda^{n+2(L-1)}\sum_{i,k=1}^{n-1}\theta_{in}\theta_{kn}\partial_i^\mu\partial_\mu^k\tilde F_{n,0,n+L-2,L}(x_1,\theta_{1n},\dots x_n)\end{aligned} \tag{4.16}$$

with

$$\begin{aligned}\tilde F_{n,0,m,\bar m}(x_1,\theta_{1n},\dots x_n) = \frac{1}{m!\bar m!}\int dS_{n+1}\dots\int dS_{n+m}\int d\bar S_1\dots\int d\bar S_{\bar m} \\ (-\bar\theta_{\bar m}^2)F_{n,0,m,\bar m}(x_1,\theta_{1n+m},\dots x_{n+m},\bar x_1,\bar\theta_{1\bar m},\dots \bar x_{\bar m})\end{aligned} \tag{4.17}$$

By differentiation with respect to the antichiral coupling the Green functions $\Gamma_{n,0,m,\bar m}$ can be related to Green functions with covariant $\bar A^3$-insertions (cf. (2.16)). In particular $\tilde F_{n,0,n+L-2,L}$ (4.16) may be expressed in terms of Green functions having an integrated $\bar\varphi^3$-insertion on an internal vertex and otherwise interaction vertices of the Wess-Zumino model. Similarly R'-symmetry makes it possible, to express chiral Green functions of the Wess-Zumino model directly in terms of Green functions with an antichiral integrated vertex insertion. In summary, the results of the algebraic analysis of non-renormalization theorems is expressed in the following simple formulae in momentum space:

$$\begin{aligned}&\Gamma_n^{(L)}(p_1,\theta_{1n},\dots p_n) \\ &= \frac{\lambda}{L}\Big(\big[\int d^4x\;\bar Q_4(x)\big]_4\cdot\Gamma\Big)_n^{(L)}(p_1,\theta_{1n},\dots p_n) \\ &= \frac{\lambda}{L}\Big(\big[-\frac{1}{4}\int d^4x\;(\delta_{\dot\alpha}^{\bar Q}\delta^{\bar Q\dot\alpha}\bar Q_3(x))\big]_4\cdot\Gamma\Big)_n^{(L)}(p_1,\theta_{1n},\dots p_n) \\ &= \frac{\lambda}{L}\sum_{i,k=1}^{n-1}\theta_{in}\cdot\theta_{kn}\;p_k^\mu p_{\mu i}\Big(\big[\int d^4x\;\bar Q_3(x)\big]_3\cdot\Gamma\Big)_n^{(L)}(p_1,\theta_{1n},\dots p_n)\end{aligned} \tag{4.18}$$

The insertions are defined by the action principle, which reads in components ($\bar\Lambda = \bar\lambda(x)+\bar\theta\bar\chi(x)+\bar\theta^2\bar f(x)$):

$$\begin{aligned}\lim_{\Lambda,\bar\Lambda\to\lambda}\frac{\delta}{\delta\bar\lambda(x)}\Gamma(A,\bar A,\Lambda,\bar\Lambda) &= [\bar Q_4(x)]_4\cdot\Gamma(A,\bar A) \\ &= \big[-\frac{1}{4}(\bar F\bar\varphi^2-\frac{1}{2}\bar\psi^2\bar\varphi)+\mathcal{O}(\hbar)\big]_4\cdot\Gamma(A,\bar A)\end{aligned} \tag{4.19}$$

$$\begin{aligned}\lim_{\Lambda,\bar\Lambda\to\lambda}\frac{\delta}{\delta\bar f(x)}\Gamma(A,\bar A,\Lambda,\bar\Lambda) &= [\bar Q_3(x)]_3\cdot\Gamma(A,\bar A) \\ &= \big[-\frac{1}{12}\bar\varphi^3+\mathcal{O}(\hbar)\big]_3\cdot\Gamma(A,\bar A)\end{aligned} \tag{4.20}$$

In the BPHZ scheme the insertions are determined by Γ_{eff}. Using the general normalization conditions for the wave-function normalization (4.5) the insertions are given by

$$
\begin{aligned}
\bar{Q}_4(x) &= \lim_{\Lambda,\bar{\Lambda}\to\lambda} \frac{\delta}{\delta\bar{\lambda}(x)}\Gamma_{eff} && (4.21)\\
&= -\frac{1}{4}(\bar{F}\bar{\varphi}^2 - \frac{1}{2}\bar{\psi}^2\bar{\varphi}) + \sum_{L=1} z^{(L)} L\lambda^{2L-1}(\bar{F}F - \frac{i}{2}\partial^\mu\psi\sigma\bar{\psi} - \bar{\varphi}\Box\varphi)\\
\bar{Q}_3(x) &= \lim_{\Lambda,\bar{\Lambda}\to\lambda} \frac{\delta}{\delta\bar{f}(x)}\Gamma_{eff} && (4.22)\\
&= -\frac{1}{12}\bar{\varphi}^3 + \sum_{L=1}^{\infty} L z^{(L)}\lambda^{2L-1}F\bar{\varphi}
\end{aligned}
$$

In passing we notice that the counterterms to the kinetic term have the same algebraic properties as the interaction vertex (2.7). They can be expressed as second variations of supersymmetry transformations or as their complex conjugate:

$$
\bar{F}F + \frac{i}{2}\psi\sigma\partial\bar{\psi} - \varphi\Box\bar{\varphi} = \frac{1}{4}\delta^Q_\alpha\delta^{Q\alpha}\bar{F}\varphi \tag{4.23a}
$$

$$
\bar{F}F - \frac{i}{2}\partial\psi\sigma\bar{\psi} - \bar{\varphi}\Box\varphi = \frac{1}{4}\delta^{\bar{Q}\dot{\alpha}}\delta^{\bar{Q}}_{\dot{\alpha}}F\ \bar{\varphi} \tag{4.23b}
$$

In the algebraic analysis of the non-renormalization theorems counterterms to the kinetic term appear in the same way as the interaction vertices: Insertions of the supersymmetric invariant are transformed into the lowest component of the respective supermultiplet by extracting at the same time two derivatives on the external legs. This implies that the full insertion related to the derivative of the antichiral coupling can be written as a second supersymmetry variation of a lower dimensional field monomial.

5. Non-renormalization theorems in the massless model

The algebraic analysis of non-renormalization theorems in the massless model is carried out in the same way as in the massive one. The renormalization of the massless Wess–Zumino model with local field couplings can be performed by the massless version of the BPHZ scheme, the BPHZL scheme [14] in superspace [13]. In addition to the ultraviolet degree of power counting one assigns to every field also an infrared degree of power counting

$$
\dim^{IR} A = \dim^{IR} \bar{A} = 1 \quad \text{and} \quad \dim^{IR}\Lambda = \dim^{IR}\bar{\Lambda} = 0 \tag{5.1}
$$

The Γ_{eff} will only include counterterms with infrared dimension greater or equal 4 (cf. (4.1),(4.2) and (4.4)):

$$\Gamma_{eff} = \Gamma_o + \Gamma_{int} + \Gamma_{ct} \tag{5.2}$$

with

$$\begin{aligned}
\Gamma_o &= \int dV\ A e^{2i\theta\sigma\bar\theta\partial}\bar A + \frac{M(s-1)}{8}\Big(\int dS\ A^2 + \int d\bar S\ \bar A^2\Big) \qquad (5.3)\\
\Gamma_{int} &= \frac{1}{48}\Big(\int dS\ \Lambda A^3 + \int d\bar S\ \bar\Lambda\bar A^3\Big)\\
\Gamma_{ct} &= \sum_{L=1}^{\infty}\int dV\ z^{(L)}\Lambda^L A e^{2i\theta\sigma\bar\theta\partial}(\bar\Lambda^L\bar A)
\end{aligned}$$

$M(s-1)$ is the auxiliary mass term of the BPHZL scheme with the subtraction parameter s. The massless limit is achieved by taking $s=1$ after all subtractions have been performed (see e.g. [6] for a detailed discussion of the BPHZL scheme in superspace). The massless model satisfies the supersymmetry Ward identities (2.11) and its Green functions are R'-symmetric (cf. (3.2) and (3.10)):

$$\mathcal{W}^Q_\alpha\Gamma = 0, \qquad \mathcal{W}^{\bar Q}_{\dot\alpha}\Gamma = 0 \quad \text{and} \quad \mathcal{W}^{R'}\Gamma = 0. \tag{5.4}$$

The normalization condition of the wave-function normalization has to be taken at a non-zero normalization point κ^2 (4.5). It is well-known that the massless Wess–Zumino model is invariant under conformal R-symmetry. For this reason the only non-vanishing chiral 1PI Green functions are those with three external chiral A-legs. The construction of chiral 3-point functions in expressions of Green functions of the Wess–Zumino model with local couplings proceeds as in the massive model and chiral 3-point functions are related to superficially convergent Green functions with covariant vertex insertions. Taking into account the infrared power counting degree of the insertions, eq. (4.18) can be taken without modifications to the massless model:

$$\begin{aligned}
&\Gamma_3^{(L)}(p_1,\theta_{13},\dots p_3) \qquad (5.5)\\
&= \frac{\lambda}{L}\Big(\Big[\int d^4x\ \bar Q_4(x)\Big]^4_4\cdot\Gamma\Big)_3(p_1,\theta_{13},\dots p_3)\\
&= \frac{\lambda}{L}\Big(\Big[-\frac{1}{4}\int d^4x\ (\delta^{\bar Q}_{\dot\alpha}\delta^{\bar Q\dot\alpha}\bar Q_3(x))\Big]^4_4\cdot\Gamma\Big)_3(p_1,\theta_{13},\dots p_3)\\
&= \frac{\lambda}{L}\sum_{i,k=1}^{2}\theta_{i3}\theta_{k3}p_kp_i\Big(\Big[\int d^4x\ \bar Q_3(x)\Big]^3_3\cdot\Gamma)_3(p_1,\theta_{13},\dots p_3)
\end{aligned}$$

The insertions $\bar Q_4$ and $\bar Q_3$ are defined by differentiation with respect to the local antichiral coupling (see eqs. (4.19) and (4.20)). In the expressions of eq. (5.5) there are no off-shell infrared problems, since Green functions with a single integrated 3-3-insertion exist for non-exceptional momenta. Therefore the ultraviolet degree of power counting is improved as in the massive model by relating the invariant vertices to the lower dimensional field monomials, and neither an ultraviolet nor an infrared overall subtraction has to be performed (cf. (4.15)):

$$d_\Gamma = r_\Gamma = -2 \quad \text{for chiral 3-point functions.} \tag{5.6}$$

In contrast to the massive model the conclusion, that chiral Green functions vanish at zero momentum, is not true in the massless model, since the Green functions with integrated $\bar{Q}_3$-insertions have poles in the external momenta. In the above expressions (5.5) these poles are canceled by multiplication with the external momenta resulting in a non-vanishing and even in higher than 2-loop order logarithmically divergent contribution to the effective potential at zero momentum. A detailed discussion of the infrared behavior is most conveniently done in terms of component fields. We take equation (A.4) with $p_3 = 0$:

$$\begin{aligned}\Gamma^{(L)}_{F\varphi\varphi}(p_1, -p_1, 0) = \frac{\lambda}{L} \lim_{p_3\to 0}\Bigg(& p_3^2\Big(\big[\int d^4x\, \bar{Q}_3(x)\big]^3_3 \cdot \Gamma\Big)^{\alpha}_{\alpha F}(p_1, p_2, p_3) \\ & + p_1^2\Big(\big[\int d^4x\, \bar{Q}_3(x)\big]^3_3 \cdot \Gamma\Big)^{\alpha}_{\alpha F}(p_1, p_3, p_2) \\ & + p_1^2\Big(\big[\int d^4x\, \bar{Q}_3(x)\big]^3_3 \cdot \Gamma\Big)^{\alpha}_{\alpha F}(p_2, p_3, p_1)\Bigg)\end{aligned} \tag{5.7}$$

By infrared power counting we conclude, that the chiral Green function of the Wess–Zumino model on the left-hand-side exists, since the external φ-leg, which is associated with p_3, has infrared dimension 1. In the lowest contributing loop order $L = 2$ there are no divergent subdiagrams and the expression is finite according to the improved power counting formula (5.6). For this reason and for dimensional reasons the value of the chiral 3-point function at $p_3 = 0$ is a pure number in 2-loop order and exists therefore also at the exceptional momentum $p_1 = p_3 = 0$. In higher orders self-energy diagrams appear as subdiagrams and have to be subtracted. Then the chiral 3-point function depends on the ratio $\frac{p_1^2}{\kappa^2}$ and is in general logarithmically infrared divergent at $p_1 = 0$:

$$\begin{aligned}&\Gamma^{(2)}_{F\varphi\varphi}(p_1, -p_1, 0) = C \\ &\Gamma^{(L)}_{F\varphi\varphi}(p_1, -p_1, 0) = f^{(L)}(p_1^2\kappa^2) \quad \text{for} \qquad L \geq 3\end{aligned} \tag{5.8}$$

The Green functions with the 3-3-insertions appearing on the right-hand-side of eq. (5.7) are in general infrared divergent for $p_3 = 0$, since one produces in this way a second integrated 3-3- or even 2-2-insertion. For dimensional reasons they have the form

$$\Big(\big[\int d^4x\, \bar{Q}_3(x)\big]^3_3 \cdot \Gamma\Big)^{\alpha}_{\alpha F}(p_1, p_2, p_3) = \sum_{i=1}^{3} \frac{1}{p_i^2} g_i\Big(\frac{p_1^2}{\kappa^2}, \frac{p_2^2}{\kappa^2}, \frac{p_3^2}{\kappa^2}\Big) \tag{5.9}$$

with the functions g_i being at most logarithmically divergent at $p_i = 0$. If the vanishing momentum is associated with the external F-leg as it happens in the first line of eq. (5.7), one produces in principle an integrated 2-2-insertion, which corresponds to a 2-dimensional mass insertion into a massless diagram. The corresponding integrals do not exist and the Green function in (5.9) has a pole even for $p_1 \neq 0$. In its contribution to the chiral Green functions of the Wess–Zumino model the pole is canceled by multiplication with p_3^2. In the second and third line of eq. (5.7) the momenta associated with the ψ_α-legs are put to zero. From the existence of the left-hand-side at $p_3 = 0$ we can conclude that the corresponding Green functions with integrated $\bar{Q}_3$-insertion exist for $p_1 \neq 0$:

$$\Big([\int d^4x \; \bar{Q}_3(x)]^3_3 \cdot \Gamma\Big)^{\alpha}_{\alpha F}(p_1, 0, -p_1) = \frac{1}{p_1^2} g(\frac{p_1^2}{\kappa^2}). \tag{5.10}$$

At zero momentum all poles are canceled by the additional external momentum factors leaving behind a non-vanishing finite number in 2-loop order and a logarithmically divergent expression in orders $L \geq 3$.

Since in 2-loop order the chiral 3-point function is a finite integral, its value at $p_i = 0$, which has been calculated in Ref. [11], is a characteristic number of the Wess–Zumino model.

6. Conclusions

The algebraic property of interaction vertices to be second supersymmetry variations of lower dimensional field monomials has been used to derive the non-renormalization theorems on the basis of algebraic renormalization. The technical tool for carrying out the analysis is the extension of the ordinary coupling of the Wess-Zumino model to an external field multiplet in superspace. Local couplings in the Wess-Zumino model turned out to be a useful technique for getting insight into the chiral/antichiral vertex structure of different diagrams: Based on a modified R-symmetry we could prove, that all non-local vertex functions consist of a definite number of chiral and antichiral vertices. Green functions with only chiral external A-legs are superficially convergent by power counting in the extended model. Using the supersymmetry Ward identities explicit expressions for the chiral vertex functions have been derived by extracting two supersymmetry variations from an internal antichiral vertex and transforming them to two derivatives acting on external legs. By performing this analysis in the massless version the cancellation of ultraviolet divergencies in chiral vertex functions can be proven without referring to properties of the superpotential inflicted with infrared singularities at zero momentum.

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Appendix: Non-renormalization theorems in component fields

In this appendix we give the expressions for the non-renormalization of chiral vertex functions, eq. (4.18), in terms of component fields. We start with the 2-point function. The contributing diagrams have L chiral and L antichiral vertices and the function $\tilde{F}_{2,0,L,L}$ (4.17) does not depend on θ due to R'-symmetry (cf. section 3 and in particular eq. (3.10)):

$$\tilde{F}_{2,0,L,L}(x_1, x_2, \theta_{12}) = f(x_1, x_2) = f(x_2, x_1) \tag{A.1}$$

One finds in momentum space:

$$\Gamma^{(L)}_{F\varphi}(p_1, -p_1) = \frac{\lambda}{L} p_1^2 \Big(\big[\int d^4x \; \bar{Q}_3(x) \big]_3 \cdot \Gamma \Big)^{(L)}_{FF}(p_1, -p_1) \tag{A.2}$$

with $\bar{Q}_3$ being defined by differentiation with respect to the highest component $\bar{f}(x)$ of the antichiral coupling (see eq. (4.20)).

For the 3-point function one has $1 + L$ chiral and L antichiral vertices in every loop order and the function $\tilde{F}_{3,0,1+L,L}(x_1, x_2, \theta_{12}, \theta_{13})$ (4.17) is of degree 2 in θ_{i3}. After having properly symmetrized we find

$$\tilde{F}_{3,0,L+1,L}(x_1, x_2, x_3, \theta_{13}, \theta_{12}) = \theta_{12}^2 f(x_1, x_2; x_3) + \theta_{13}^2 f(x_1, x_3; x_2) + \theta_{23}^2 f(x_2, x_3; x_1) \tag{A.3}$$

with $\theta_{23} = \theta_{13} - \theta_{12}$ and $f(x_1, x_2; x_3) = f(x_2, x_1; x_3)$. Inserting $\tilde{F}_{3,0,L+1,L}$ in this form into the generating functional of 1PI Green functions one derives for the 3-point function of component fields the following expression ($p_1 + p_2 + p_3 = 0$):

$$\begin{aligned}\Gamma^{(L)}_{F\varphi\varphi}(p_1, p_2, p_3) = \frac{\lambda}{L} \epsilon^{\alpha\beta} \bigg(& p_3^2 \Big(\big[\int d^4x \; \bar{Q}_3(x) \big] \cdot \Gamma \Big)^{(L)}_{\beta\alpha F}(p_1, p_2, p_3) \\ & + p_2^2 \Big(\big[\int d^4x \; \bar{Q}_3(x) \big] \cdot \Gamma \Big)^{(L)}_{\beta\alpha F}(p_1, p_3, p_2) \\ & + p_1^2 \Big(\big[\int d^4x \; \bar{Q}_3(x) \big] \cdot \Gamma \Big)^{(L)}_{\beta\alpha F}(p_2, p_3, p_1) \bigg)\end{aligned} \tag{A.4}$$

From supersymmetry one has

$$\Gamma_{\alpha\beta\varphi}(p_1, p_2, p_3) = \frac{1}{2} \epsilon_{\alpha\beta} \Gamma_{F\varphi\varphi}(p_1, p_2, p_3) \tag{A.5}$$

and

$$\Big([\int d^4x\;\bar{Q}_3(x)]\cdot\Gamma\Big)_{FF\varphi}(p_1,p_2,p_3) = \Big([\int d^4x\;\bar{Q}_3(x)]\cdot\Gamma\Big)^{\alpha}_{\;\;\alpha F}(p_1,p_3,p_2) + \Big([\int d^4x\;\bar{Q}_3(x)]\cdot\Gamma\Big)^{\alpha}_{\;\;\alpha F}(p_2,p_3,p_1) \tag{A.6}$$

The Green functions with external fermion legs are defined by:

$$\cdots\frac{\delta}{\delta\psi^{\beta}(x_2)}\frac{\delta}{\delta\psi^{\alpha}(x_1)}\Gamma\Big|_{\text{fields}=0} = \Gamma_{\alpha\beta\ldots}(x_1,x_2,\ldots) \tag{A.7}$$

For the general n-point functions the respective equations are more complicated, since one finds two supersymmetric functions of degree $2(n-2)$ in θ_{in}, which contribute to $\tilde{F}_{n,0,n+L-2,L}$:

$$\tilde{F}_{n,0,n+L-2,L}(x_1,\theta_{1n},\ldots x_n) = \Big(\prod_{i=2}^{n}\theta_{in}^2 f_1(x_1;x_2,\ldots x_n) + (n-1)\text{ perm.}\Big) + \Big(\theta_{1n}\cdot\theta_{2n}\prod_{i=3}^{n}\theta_{in}^2 f(x_1,x_2;x_3,\ldots x_n) + \;(\tfrac{n(n-1)}{2}-1)\text{ perm.}\Big) \tag{A.8}$$

and one gets for the chiral n-point functions the result:

$$\begin{aligned}&\Gamma^{(L)}_{F\varphi\ldots\varphi}(p_1,p_2,\ldots p_n) \qquad\qquad (\text{A.9})\\ &= \frac{\lambda}{L}\Big(\sum_{k=1}^{n-1}p_k^2\Big([\int d^4x\;\bar{Q}_3(x)]_3\cdot\Gamma\Big)^{(L)}_{FF\varphi\ldots\varphi}(p_k,p_n;p_1,\ldots\hat{p}_k,\ldots p_{n-1})\\ &\quad + 2\epsilon^{\alpha\beta}\sum_{k=1}^{n-2}\sum_{i=k+1}^{n-1}p_k^{\mu}p_{\mu i}\Big([\int d^4x\;\bar{Q}_3(x)]_3\cdot\Gamma\Big)^{(L)}_{\beta\alpha F\varphi\ldots\varphi}(p_k,p_i;p_n,p_1,\ldots\hat{p}_k,\ldots\hat{p}_i,\ldots p_{n-1})\Big)\end{aligned}$$

The notation "$\hat{p}_i$" indicates here that these momenta are omitted.

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